# 43

## NUCLEAR PHYSICS

**43.1.** **IDENTIFY and SET UP:** The pre-subscript is $Z$, the number of protons. The pre-superscript is the mass number $A$. $A = Z + N$, where $N$ is the number of neutrons.

**EXECUTE:** **(a)** ${}^{28}_{14}\text{Si}$ has 14 protons and 14 neutrons.

**(b)** ${}^{85}_{37}\text{Rb}$ has 37 protons and 48 neutrons.

**(c)** ${}^{205}_{81}\text{Tl}$ has 81 protons and 124 neutrons.

**EVALUATE:** The number of protons determines the chemical element.

**43.2.** **IDENTIFY:** Calculate the spin magnetic energy shift for each spin component. Calculate the energy splitting between these states and relate this to the frequency of the photons.

**(a) SET UP:** From Example 43.2, when the $z$-component of $\vec{S}$ (and $\vec{\mu}$) is parallel to $\vec{B}$, $U = -|\mu_z|B = -2.7928\mu_n B$. When the $z$-component of $\vec{S}$ (and $\vec{\mu}$) is antiparallel to $\vec{B}$, $U = +|\mu_z|B = +2.7928\mu_n B$. The state with the proton spin component parallel to the field lies lower in energy. The energy difference between these two states is $\Delta E = 2(2.7928\mu_n B)$.

**EXECUTE:** $\Delta E = hf$ so $f = \dfrac{\Delta E}{h} = \dfrac{2(2.7928\mu_n)B}{h} = \dfrac{2(2.7928)(5.051\times10^{-27}\ \text{J/T})(1.65\ \text{T})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}}$

$f = 7.03\times10^{7}\ \text{Hz} = 70.3\ \text{MHz}$

And then $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\ \text{m/s}}{7.03\times10^{7}\ \text{Hz}} = 4.26\ \text{m}$

**EVALUATE:** From Figure 32.4 in the textbook, these are radio waves.

**(b) SET UP:** From Eqs. (27.27) and (41.40) and Figure 41.18 in the textbook, the state with the $z$-component of $\vec{\mu}$ parallel to $\vec{B}$ has lower energy. But, since the charge of the electron is negative, this is the state with the electron spin component antiparallel to $\vec{B}$. That is, for $m_s = -\frac{1}{2}$, the state lies lower in energy.

**EXECUTE:** For the $m_s = +\frac{1}{2}$ state,

$$U = +(2.00232)\left(\frac{e}{2m}\right)\left(+\frac{\hbar}{2}\right)B = +\tfrac{1}{2}(2.00232)\left(\frac{e\hbar}{2m}\right)B = +\tfrac{1}{2}(2.00232)\mu_B B.$$

For the $m_s = -\frac{1}{2}$ state, $U = -\frac{1}{2}(2.00232)\mu_B B$. The energy difference between these two states is $\Delta E = (2.00232)\mu_B B$.

$$\Delta E = hf \text{ so } f = \frac{\Delta E}{h} = \frac{2.00232\mu_B B}{h} = \frac{(2.00232)(9.274\times10^{-24}\ \text{J/T})(1.65\ \text{T})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 4.62\times10^{10}\ \text{Hz} = 46.2\ \text{GHz}.$$

And $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\ \text{m/s}}{4.62\times10^{10}\ \text{Hz}} = 6.49\times10^{-3}\ \text{m} = 6.49\ \text{mm}.$

**EVALUATE:** From Figure 32.4 in the textbook, these are microwaves. The interaction energy with the magnetic field is inversely proportional to the mass of the particle, so it is less for the proton than for the electron. The smaller transition energy for the proton produces a larger wavelength.

**43.3.** **IDENTIFY:** Calculate the spin magnetic energy shift for each spin state of the 1*s* level. Calculate the energy splitting between these states and relate this to the frequency of the photons.

**SET UP:** When the spin component is parallel to the field the interaction energy is $U = -\mu_z B.$ When the spin component is antiparallel to the field the interaction energy is $U = +\mu_z B.$ The transition energy for a transition between these two states is $\Delta E = 2\mu_z B,$ where $\mu_z = 2.7928\mu_{\text{n}}.$ The transition energy is related to the photon frequency by $\Delta E = hf,$ so $2\mu_z B = hf.$

**EXECUTE:** $B = \dfrac{hf}{2\mu_z} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(22.7\times10^{6}\text{ Hz})}{2(2.7928)(5.051\times10^{-27}\text{ J/T})} = 0.533\text{ T}$

**EVALUATE:** This magnetic field is easily achievable. Photons of this frequency have wavelength $\lambda = c/f = 13.2\text{ m}.$ These are radio waves.

**43.4.** **IDENTIFY:** The interaction energy of the nuclear spin angular momentum with the external field is $U = -\mu_z B.$ The transition energy $\Delta E$ for the neutron is related to the frequency and wavelength of the photon by $\Delta E = hf = \dfrac{hc}{\lambda}.$

**SET UP:** $|\mu_z| = 1.9130\mu_{\text{n}},$ where $\mu_{\text{n}} = 3.15245\times10^{-8}\text{ eV/T}.$

**EXECUTE:** **(a)** As in Example 43.2, $\Delta E = 2(1.9130)(3.15245\times10^{-8}\text{ eV/T})(2.30\text{ T}) = 2.77\times10^{-7}\text{ eV}.$ Since $\vec{\mu}$ and $\vec{S}$ are in opposite directions for a neutron, the antiparallel configuration is lower energy. This result is smaller than but comparable to that found in the example for protons.

**(b)** $f = \dfrac{\Delta E}{h} = 66.9\text{ MHz},\ \lambda = \dfrac{c}{f} = 4.48\text{ m}.$

**EVALUATE:** $\Delta E$ and *f* for neutrons are smaller than the corresponding values for protons that were calculated in Example 43.2.

**43.5.** **(a) IDENTIFY:** Find the energy equivalent of the mass defect.

**SET UP:** A $^{11}_{5}\text{B}$ atom has 5 protons, $11-5=6$ neutrons, and 5 electrons. The mass defect therefore is $\Delta M = 5m_{\text{p}} + 6m_{\text{n}} + 5m_{\text{e}} - M(^{11}_{5}\text{B}).$

**EXECUTE:** $\Delta M = 5(1.0072765\text{ u}) + 6(1.0086649\text{ u}) + 5(0.0005485799\text{ u}) - 11.009305\text{ u} = 0.08181\text{ u}.$ The energy equivalent is $E_{\text{B}} = (0.08181\text{ u})(931.5\text{ MeV/u}) = 76.21\text{ MeV}.$

**(b) IDENTIFY** and **SET UP:** Eq. (43.11): $E_{\text{B}} = C_1 A - C_2 A^{2/3} - C_3 Z(Z-1)/A^{1/3} - C_4(A-2Z)^2/A$

The fifth term is zero since *Z* is odd but *N* is even. $A = 11$ and $Z = 5.$

**EXECUTE:** $E_{\text{B}} = (15.75\text{ MeV})(11) - (17.80\text{ MeV})(11)^{2/3} - (0.7100\text{ MeV})5(4)/11^{1/3} - (23.69\text{ MeV})(11-10)^2/11.$

$E_{\text{B}} = +173.25\text{ MeV} - 88.04\text{ MeV} - 6.38\text{ MeV} - 2.15\text{ MeV} = 76.68\text{ MeV}$

The percentage difference between the calculated and measured $E_{\text{B}}$ is $\dfrac{76.68\text{ MeV} - 76.21\text{ MeV}}{76.21\text{ MeV}} = 0.6\%.$

**EVALUATE:** Eq. (43.11) has a greater percentage accuracy for $^{62}\text{Ni}.$ The semi-empirical mass formula is more accurate for heavier nuclei.

**43.6.** **IDENTIFY:** The mass defect is the total mass of the constituents minus the mass of the atom.

**SET UP:** 1 u is equivalent to 931.5 MeV. $^{238}_{92}\text{U}$ has 92 protons, 146 neutrons and 238 nucleons.

**EXECUTE:** **(a)** $146m_{\text{n}} + 92m_{\text{H}} - m_{\text{U}} = 1.93\text{ u}.$

**(b)** $1.80\times10^{3}\text{ MeV}.$

**(c)** $7.56\text{ MeV}$ per nucleon (using $931.5\text{ MeV/u}$ and 238 nucleons).

**EVALUATE:** The binding energy per nucleon we calculated agrees with Figure 43.2 in the textbook.

**43.7.** **IDENTIFY** and **SET UP:** The text calculates that the binding energy of the deuteron is 2.224 MeV. A photon that breaks the deuteron up into a proton and a neutron must have at least this much energy. $E=\dfrac{hc}{\lambda}$ so $\lambda=\dfrac{hc}{E}$

**EXECUTE:** $\lambda=\dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{2.224\times10^{6}\text{ eV}}=5.575\times10^{-13}\text{ m}=0.5575\text{ pm}.$

**EVALUATE:** This photon has gamma-ray wavelength.

**43.8.** **IDENTIFY:** The binding energy of the nucleus is the energy of its constituent particles minus the energy of the carbon-12 nucleus.

**SET UP:** In terms of the masses of the particles involved, the binding energy is

$$E_{\text{B}}=(6m_{\text{H}}+6m_{\text{n}}-m_{\text{C}-12})c^2.$$

**EXECUTE:** **(a)** Using the values from Table 43.2, we get

$$E_{\text{B}}=[6(1.007825\text{ u})+6(1.008665\text{ u})-12.000000\text{ u})](931.5\text{ MeV/u})=92.16\text{ MeV}$$

**(b)** The binding energy per nucleon is $(92.16\text{ MeV})/(12\text{ nucleons})=7.680\text{ MeV/nucleon}$

**(c)** The energy of the C-12 nucleus is $(12.0000\text{ u})(931.5\text{ MeV/u})=11178\text{ MeV}.$ Therefore the percent of the mass that is binding energy is $\dfrac{92.16\text{ MeV}}{11178\text{ MeV}}=0.8245\%.$

**EVALUATE:** The binding energy of 92.16 MeV binds 12 nucleons. The binding energy per nucleon, rather than just the total binding energy, is a better indicator of the strength with which a nucleus is bound.

**43.9.** **IDENTIFY:** Conservation of energy tells us that the initial energy (photon plus deuteron) is equal to the energy after the split (kinetic energy plus energy of the proton and neutron). Therefore the kinetic energy released is equal to the energy of the photon minus the binding energy of the deuteron.

**SET UP:** The binding energy of a deuteron is 2.224 MeV and the energy of the photon is $E=hc/\lambda$. Kinetic energy is $K=\frac{1}{2}mv^2$.

**EXECUTE:** **(a)** The energy of the photon is

$$E_{\text{ph}}=\frac{hc}{\lambda}=\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{3.50\times10^{-13}\text{ m}}=5.68\times10^{-13}\text{ J}.$$

The binding of the deuteron is $E_{\text{B}}=2.224\text{ MeV}=3.56\times10^{-13}\text{ J}.$ Therefore the kinetic energy is $K=(5.68-3.56)\times10^{-13}\text{ J}=2.12\times10^{-13}\text{ J}=1.32\text{ MeV}.$

**(b)** The particles share the energy equally, so each gets half. Solving the kinetic energy for $v$ gives

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(1.06\times10^{-13}\text{ J})}{1.6605\times10^{-27}\text{ kg}}}=1.13\times10^{7}\text{ m/s}$$

**EVALUATE:** Considerable energy has been released, because the particle speeds are in the vicinity of the speed of light.

**43.10.** **IDENTIFY:** The mass defect is the total mass of the constituents minus the mass of the atom.

**SET UP:** 1 u is equivalent to 931.5 MeV. ${}^{14}_{7}\text{N}$ has 7 protons and 7 neutrons. ${}^{4}_{2}\text{He}$ has 2 protons and 2 neutrons.

**EXECUTE:** **(a)** $7(m_{\text{n}}+m_{\text{H}})-m_{\text{N}}=0.112\text{ u},$ which is 105 MeV, or 7.48 MeV per nucleon.

**(b)** Similarly, $2(m_{\text{H}}+m_{\text{n}})-m_{\text{He}}=0.03038\text{ u}=28.3\text{ MeV},$ or 7.07 MeV per nucleon.

**EVALUATE:** **(c)** The binding energy per nucleon is a little less for ${}^{4}_{2}\text{He}$ than for ${}^{14}_{7}\text{N}$. This is in agreement with Figure 43.2 in the textbook.

**43.11.** **IDENTIFY:** Use Eq. (43.11) to calculate the binding energy of two nuclei, and then calculate their binding energy per nucleon.

**SET UP** and **EXECUTE:** ${}^{86}_{36}\text{Kr}$: $A=86$ and $Z=36$. $N=A-Z=50,$ which is even, so for the last term in Eq. (43.11) we use the plus sign. Putting the given number in the equation and using the values for the constants given in the textbook, we have

$$E_B = (15.75\text{ MeV})(86) - (17.80\text{ MeV})(86)^{2/3} - (0.71\text{ MeV})\frac{(36)(35)}{86^{1/3}}$$
$$-(23.69\text{ MeV})\frac{(86-72)^2}{86} + (39\text{ MeV})(86)^{-4/3}.$$

$E_B = 751.1\text{ MeV}$ and $\frac{E_B}{A} = 8.73\text{ MeV/nucleon.}$

${}^{180}_{73}\text{Ta}$: $A = 180, Z = 73, N = 180 - 73 = 107$, which is odd.

$$E_B = (15.75\text{ MeV})(180) - (17.80\text{ MeV})(180)^{2/3} - (0.71\text{ MeV})\frac{(73)(72)}{180^{1/3}}$$
$$-(23.69\text{ MeV})\frac{(180-146)^2}{180} + (39\text{ MeV})(180)^{-4/3}$$

$E_B = 1454.4\text{ MeV}$ and $\frac{E_B}{A} = 8.08\text{ MeV/nucleon.}$

**EVALUATE:** The binding energy per nucleon is less for ${}^{180}_{73}\text{Ta}$ than for ${}^{86}_{36}\text{Kr}$, in agreement with Figure 43.2.

**43.12.** **IDENTIFY:** Compare the total mass on each side of the reaction equation. Neglect the masses of the neutrino and antineutrino.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** **(a)** The energy released is the energy equivalent of $m_n - m_p - m_e = 8.40\times10^{-4}$ u, or 783 keV.

**(b)** $m_n > m_p$, and the decay is not possible.

**EVALUATE:** $\beta^-$ and $\beta^+$ particles have the same mass, equal to the mass of an electron.

**43.13.** **IDENTIFY:** In each case determine how the decay changes $A$ and $Z$ of the nucleus. The $\beta^+$ and $\beta^-$ particles have charge but their nucleon number is $A = 0$.

**(a) SET UP:** $\alpha$-decay: $Z$ increases by 2, $A = N + Z$ decreases by 4 (an $\alpha$ particle is a ${}^4_2\text{He}$ nucleus)

**EXECUTE:** ${}^{239}_{94}\text{Pu} \to {}^4_2\text{He} + {}^{235}_{92}\text{U}$

**(b) SET UP:** $\beta^-$ decay: $Z$ increases by 1, $A = N + Z$ remains the same (a $\beta^-$ particle is an electron, ${}^{0}_{-1}\text{e}$)

**EXECUTE:** ${}^{24}_{11}\text{Na} \to {}^{0}_{-1}\text{e} + {}^{24}_{12}\text{Mg}$

**(c) SET UP:** $\beta^+$ decay: $Z$ decreases by 1, $A = N + Z$ remains the same (a $\beta^+$ particle is a positron, ${}^{0}_{+1}\text{e}$)

**EXECUTE:** ${}^{15}_{8}\text{O} \to {}^{0}_{+1}\text{e} + {}^{15}_{7}\text{N}$

**EVALUATE:** In each case the total charge and total number of nucleons for the decay products equals the charge and number of nucleons for the parent nucleus; these two quantities are conserved in the decay.

**43.14.** **IDENTIFY:** The energy released is equal to the mass defect of the initial and final nuclei.

**SET UP:** The mass defect is equal to the difference between the initial and final masses of the constituent particles.

**EXECUTE:** **(a)** The mass defect is 238.050788 u – 234.043601 u – 4.002603 u = 0.004584 u. The energy released is (0.004584 u)(931.5 MeV/u) = 4.270 MeV.

**(b)** Take the ratio of the two kinetic energies, using the fact that $K = p^2/2m$:

$$\frac{K_{Th}}{K_\alpha} = \frac{\frac{p_{Th}^2}{2m_{Th}}}{\frac{p_\alpha^2}{2m_\alpha}} = \frac{m_\alpha}{m_{Th}} = \frac{4}{234}.$$

The kinetic energy of the Th is

$$K_{Th} = \frac{4}{234+4}K_{\text{Total}} = \frac{4}{238}(4.270\text{ MeV}) = 0.07176\text{ MeV} = 1.148\times10^{-14}\text{ J}$$

Solving for $v$ in the kinetic energy gives

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(1.148\times10^{-14}\text{ J})}{(234.043601)(1.6605\times10^{-27}\text{ kg})}}=2.431\times10^{5}\text{ m/s}$$

**EVALUATE:** As we can see by the ratio of kinetic energies in part (b), the alpha particle will have a much higher kinetic energy than the thorium.

**43.15.** **IDENTIFY:** Compare the mass of the original nucleus to the total mass of the decay products.

**SET UP:** Subtract the electron masses from the neutral atom mass to obtain the mass of each nucleus.

**EXECUTE:** If $\beta^-$ decay of $^{14}\text{C}$ is possible, then we are considering the decay $^{14}_{6}\text{C}\to{}^{14}_{7}\text{N}+\beta^-$.

$\Delta m=M(^{14}_{6}\text{C})-M(^{14}_{7}\text{N})-m_\text{e}$

$\Delta m=(14.003242\text{ u}-6(0.000549\text{ u}))-(14.003074\text{ u}-7(0.000549\text{ u}))-0.0005491\text{ u}$

$\Delta m=+1.68\times10^{-4}$ u. So $E=(1.68\times10^{-4}\text{ u})(931.5\text{ MeV/u})=0.156\text{ MeV}=156\text{ keV}$

**EVALUATE:** In the decay the total charge and the nucleon number are conserved.

**43.16.** **IDENTIFY:** In each reaction the nucleon number and the total charge are conserved.

**SET UP:** An $\alpha$ particle has charge $+2e$ and nucleon number 4. An electron has charge $-e$ and nucleon number zero. A positron has charge $+e$ and nucleon number zero.

**EXECUTE:** **(a)** A proton changes to a neutron, so the emitted particle is a positron $(\beta^+)$.

**(b)** The number of nucleons in the nucleus decreases by 4 and the number of protons by 2, so the emitted particle is an alpha-particle.

**(c)** A neutron changes to a proton, so the emitted particle is an electron $(\beta^-)$.

**EVALUATE:** We have considered the conservation laws. We have not determined if the decays are energetically allowed.

**43.17.** **IDENTIFY:** The energy released is the energy equivalent of the difference in the masses of the original atom and the final atom produced in the capture. Apply conservation of energy to the decay products.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** **(a)** As in the example, $(0.000897\text{ u})(931.5\text{ MeV/u})=0.836\text{ MeV}$.

**(b)** $0.836\text{ MeV}-0.122\text{ MeV}-0.014\text{ MeV}=0.700\text{ MeV}$.

**EVALUATE:** We have neglected the rest mass of the neutrino that is emitted.

**43.18.** **IDENTIFY:** Determine the energy released during tritium decay.

**SET UP:** In beta decay an electron, $\text{e}^-$, is emitted by the nucleus. The beta decay reaction is $^{3}_{1}\text{H}\to\text{e}^-+{}^{3}_{2}\text{He}$. If neutral atom masses are used, $^{3}_{1}\text{H}$ includes one electron and $^{3}_{2}\text{He}$ includes two electrons. One electron mass cancels and the other electron mass in $^{3}_{2}\text{He}$ represents the emitted electron. Or, we can subtract the electron masses and use the nuclear masses. The atomic mass of $^{3}_{2}\text{He}$ is 3.016029 u.

**EXECUTE:** **(a)** The mass of the $^{3}_{1}\text{H}$ nucleus is $3.016049\text{ u}-0.000549\text{ u}=3.015500\text{ u}$. The mass of the $^{3}_{2}\text{He}$ nucleus is $3.016029\text{ u}-2(0.000549\text{ u})=3.014931\text{ u}$. The nuclear mass of $^{3}_{2}\text{He}$ plus the mass of the emitted electron is $3.014931\text{ u}+0.000549\text{ u}=3.015480\text{ u}$. This is slightly less than the nuclear mass for $^{3}_{1}\text{H}$, so the decay is energetically allowed.

**(b)** The mass decrease in the decay is $3.015500\text{ u}-3.015480\text{ u}=2.0\times10^{-5}$ u. Note that this can also be calculated as $m(^{3}_{1}\text{H})-m(^{4}_{2}\text{He})$, where atomic masses are used. The energy released is $(2.0\times10^{-5}\text{ u})(931.5\text{ MeV/u})=0.019\text{ MeV}$. The total kinetic energy of the decay products is 0.019 MeV, or 19 keV.

**EVALUATE:** The energy is not shared equally by the decay products because they have unequal masses.

**43.19.** **IDENTIFY** and **SET UP:** $T_{1/2}=\dfrac{\ln 2}{\lambda}$ The mass of a single nucleus is $124m_\text{p}=2.07\times10^{-25}$ kg. $|dN/dt|=0.350\text{ Ci}=1.30\times10^{10}\text{ Bq}$, $|dN/dt|=\lambda N$.

**EXECUTE:** $N=\dfrac{6.13\times10^{-3}\ \text{kg}}{2.07\times10^{-25}\ \text{kg}}=2.96\times10^{22};\ \lambda=\dfrac{|dN/dt|}{N}=\dfrac{1.30\times10^{10}\ \text{Bq}}{2.96\times10^{22}}=4.39\times10^{-13}\ \text{s}^{-1}.$

$T_{1/2}=\dfrac{\ln 2}{\lambda}=1.58\times10^{12}\ \text{s}=5.01\times10^{4}\ \text{y}.$

**EVALUATE:** Since $T_{1/2}$ is very large, the activity changes very slowly.

**43.20.** **IDENTIFY:** Eq. (43.17) can be written as $N=N_0 2^{-t/T_{1/2}}$.

**SET UP:** The amount of elapsed time since the source was created is roughly 2.5 years.

**EXECUTE:** The current activity is $N=(5000\ \text{Ci})2^{-(2.5\ \text{yr})/(5.271\ \text{yr})}=3600\ \text{Ci}$. The source is barely usable.

**EVALUATE:** Alternatively, we could calculate $\lambda=\dfrac{\ln(2)}{T_{1/2}}=0.132(\text{years})^{-1}$ and use Eq. 43.17 directly to obtain the same answer.

**43.21.** **IDENTIFY:** From the known half-life, we can find the decay constant, the rate of decay, and the activity.

**SET UP:** $\lambda=\dfrac{\ln 2}{T_{1/2}}$. $T_{1/2}=4.47\times10^{9}\ \text{yr}=1.41\times10^{17}\ \text{s}$. The activity is $\left|\dfrac{dN}{dt}\right|=\lambda N$. The mass of one $^{238}\text{U}$ is approximately $238m_\text{p}$. $1\ \text{Ci}=3.70\times10^{10}$ decays/s.

**EXECUTE:** **(a)** $\lambda=\dfrac{\ln 2}{1.41\times10^{17}\ \text{s}}=4.92\times10^{-18}\ \text{s}^{-1}.$

**(b)** $N=\dfrac{|dN/dt|}{\lambda}=\dfrac{3.70\times10^{10}\ \text{Bq}}{4.92\times10^{-18}\ \text{s}^{-1}}=7.52\times10^{27}$ nuclei. The mass $m$ of uranium is the number of nuclei times the mass of each one. $m=(7.52\times10^{27})(238)(1.67\times10^{-27}\ \text{kg})=2.99\times10^{3}\ \text{kg}.$

**(c)** $N=\dfrac{10.0\times10^{-3}\ \text{kg}}{238m_\text{p}}=\dfrac{10.0\times10^{-3}\ \text{kg}}{238(1.67\times10^{-27}\ \text{kg})}=2.52\times10^{22}$ nuclei.

$\left|\dfrac{dN}{dt}\right|=\lambda N=(4.92\times10^{-18}\ \text{s}^{-1})(2.52\times10^{22})=1.24\times10^{5}$ decays/s.

**EVALUATE:** Because $^{238}\text{U}$ has a very long half-life, it requires a large amount (about 3000 kg) to have an activity of a 1.0 Ci.

**43.22.** **IDENTIFY:** From the half-life and mass of an isotope, we can find its initial activity rate. Then using the half-life, we can find its activity rate at a later time.

**SET UP:** The activity $|dN/dt|=\lambda N$. $\lambda=\dfrac{\ln 2}{T_{1/2}}$. The mass of one $^{103}\text{Pd}$ nucleus is $103m_\text{p}$. In a time of one half-life the number of radioactive nuclei and the activity decrease by a factor of 2.

**EXECUTE:** **(a)** $\lambda=\dfrac{\ln 2}{T_{1/2}}=\dfrac{\ln 2}{(17\ \text{days})(24\ \text{h/day})(3600\ \text{s/h})}=4.7\times10^{-7}\ \text{s}^{-1}.$

$N=\dfrac{0.250\times10^{-3}\ \text{kg}}{103m_\text{p}}=1.45\times10^{21}.$ $|dN/dt|=(4.7\times10^{-7}\ \text{s}^{-1})(1.45\times10^{21})=6.8\times10^{14}\ \text{Bq}.$

**(b)** 68 days is $4T_{1/2}$ so the activity is $(6.8\times10^{14}\ \text{Bq})/2^4=4.2\times10^{13}\ \text{Bq}.$

**EVALUATE:** At the end of 4 half-lives, the activity rate is less than a tenth of its initial rate.

**43.23.** **IDENTIFY** and **SET UP:** As discussed in Section 43.4, the activity $A=|dN/dt|$ obeys the same decay equation as Eq. (43.17): $A=A_0e^{-\lambda t}$. For $^{14}\text{C}$, $T_{1/2}=5730$ y and $\lambda=\ln 2/T_{1/2}$ so $A=A_0e^{-(\ln 2)t/T_{1/2}}$; calculate $A$ at each $t$; $A_0=180.0$ decays/min.

**EXECUTE:** **(a)** $t=1000$ y, $A=159$ decays/min

**(b)** $t=50{,}000$ y, $A=0.43$ decays/min

**EVALUATE:** The time in part (b) is 8.73 half-lives, so the decay rate has decreased by a factor or $(\frac{1}{2})^{8.73}$.

**43.24.** **IDENTIFY** and **SET UP:** The decay rate decreases by a factor of 2 in a time of one half-life.

**EXECUTE:** **(a)** 24 d is $3T_{1/2}$ so the activity is $(375\text{ Bq})/(2^3) = 46.9\text{ Bq}$.

**(b)** The activity is proportional to the number of radioactive nuclei, so the percent is $\dfrac{17.0\text{ Bq}}{46.9\text{ Bq}} = 36.2\%$.

**(c)** ${}^{131}_{53}\text{I} \to {}^{0}_{-1}\text{e} + {}^{131}_{54}\text{Xe}$ The nucleus ${}^{131}_{54}\text{Xe}$ is produced.

**EVALUATE:** Both the activity and the number of radioactive nuclei present decrease by a factor of 2 in one half-life.

**43.25.** **IDENTIFY** and **SET UP:** Find $\lambda$ from the half-life and the number $N$ of nuclei from the mass of one nucleus and the mass of the sample. Then use Eq. (43.16) to calculate $|dN/dt|$, the number of decays per second.

**EXECUTE:** **(a)** $|dN/dt| = \lambda N$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(1.28\times10^{9}\text{ y})(3.156\times10^{7}\text{ s/1 y})} = 1.715\times10^{-17}\text{ s}^{-1}$$

The mass of one ${}^{40}\text{K}$ atom is approximately 40 u, so the number of ${}^{40}\text{K}$ nuclei in the sample is

$$N = \frac{1.63\times10^{-9}\text{ kg}}{40\text{ u}} = \frac{1.63\times10^{-9}\text{ kg}}{40(1.66054\times10^{-27}\text{ kg})} = 2.454\times10^{16}.$$

Then $|dN/dt| = \lambda N = (1.715\times10^{-17}\text{ s}^{-1})(2.454\times10^{16}) = 0.421\text{ decays/s}$

**(b)** $|dN/dt| = (0.421\text{ decays/s})(1\text{ Ci}/(3.70\times10^{10}\text{ decays/s})) = 1.14\times10^{-11}\text{ Ci}$

**EVALUATE:** The very small sample still contains a very large number of nuclei. But the half-life is very large, so the decay rate is small.

**43.26.** **IDENTIFY:** Apply Eq. (43.16) to calculate $N$, the number of radioactive nuclei originally present in the spill. Since the activity is proportional to the number of radioactive nuclei, Eq. (43.17) leads to $A = A_0 e^{-\lambda t}$, where $A$ is the activity.

**SET UP:** The mass of one ${}^{131}\text{Ba}$ nucleus is about 131 u.

**EXECUTE:** **(a)** $-\dfrac{dN}{dt} = 500\ \mu\text{Ci} = (500\times10^{-6})(3.70\times10^{10}\text{ s}^{-1}) = 1.85\times10^{7}\text{ decays/s}.$

$$T_{1/2} = \frac{\ln 2}{\lambda} \to \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(12\text{ d})(86{,}400\text{ s/d})} = 6.69\times10^{-7}\text{ s}.$$

$\dfrac{dN}{dt} = -\lambda N \Rightarrow N = \dfrac{-dN/dt}{\lambda} = \dfrac{1.85\times10^{7}\text{ decays/s}}{6.69\times10^{-7}\text{s}^{-1}} = 2.77\times10^{13}$ nuclei. The mass of this many ${}^{131}\text{Ba}$ nuclei is $m = 2.77\times10^{13}\text{ nuclei}\times(131\times1.66\times10^{-27}\text{ kg/nucleus}) = 6.0\times10^{-12}\text{ kg} = 6.0\times10^{-9}\text{ g} = 6.0\text{ ng}.$

**(b)** $A = A_0 e^{-\lambda t}$. $1\ \mu\text{Ci} = (500\ \mu\text{Ci})\, e^{-\lambda t}$. $\ln(1/500) = -\lambda t$.

$$t = -\frac{\ln(1/500)}{\lambda} = -\frac{\ln(1/500)}{6.69\times10^{-7}\text{s}^{-1}} = 9.29\times10^{6}\text{ s}\left(\frac{1\text{ d}}{86{,}400\text{ s}}\right) = 108\text{ days}.$$

**EVALUATE:** The time is about 9 half-lives and the activity after that time is $(500\ \mu\text{Ci})\left(\dfrac{1}{2}\right)^{9}$.

**43.27.** **IDENTIFY:** Apply $A = A_0 e^{-\lambda t}$ and $\lambda = \ln 2/T_{1/2}$.

**SET UP:** $\ln e^{x} = x$.

**EXECUTE:** $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$. $-\dfrac{(\ln 2)t}{T_{1/2}} = \ln(A/A_0)$.

$$T_{1/2} = -\frac{(\ln 2)t}{\ln(A/A_0)} = -\frac{(\ln 2)(4.00\text{ days})}{\ln(3091/8318)} = 2.80\text{ days}.$$

**EVALUATE:** The activity has decreased by more than half and the elapsed time is more than one half-life.

**43.28.** **IDENTIFY:** Apply Eq. (43.16), with $\lambda = \ln 2/T_{1/2}$.

**SET UP:** 1 mole of $^{226}\text{Ra}$ has a mass of 226 g. $1\text{ Ci} = 3.70\times10^{10}$ Bq.

**EXECUTE:** $\left|\dfrac{dN}{dt}\right| = \lambda N.$ $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{1620\text{ y }(3.15\times10^7\text{ s/y})} = 1.36\times10^{-11}\text{ s}^{-1}.$

$$N = 1\text{ g}\left(\frac{6.022\times10^{23}\text{atoms}}{226\text{ g}}\right) = 2.665\times10^{25}\text{ atoms.}$$

$\left|\dfrac{dN}{dt}\right| = \lambda N = (2.665\times10^{25})(1.36\times10^{-11}\text{ s}^{-1}) = 3.62\times10^{10}\text{ decays/s} = 3.62\times10^{10}$ Bq. Convert to Ci:

$$3.62\times10^{10}\text{ Bq}\left(\frac{1\text{ Ci}}{3.70\times10^{10}\text{ Bq}}\right) = 0.98\text{ Ci.}$$

**EVALUATE:** $dN/dt$ is negative, since the number of radioactive nuclei decreases in time.

**43.29.** **IDENTIFY** and **SET UP:** Apply Eq. (43.16), with $\lambda = \ln 2/T_{1/2}$. In one half-life, one half of the nuclei decay.

**EXECUTE:** **(a)** $\left|\dfrac{dN}{dt}\right| = 7.56\times10^{11}\text{ Bq} = 7.56\times10^{11}$ decays/s.

$\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{(30.8\text{ min})(60\text{ s/ min})} = 3.75\times10^{-4}\text{ s}^{-1}.$ $N_0 = \dfrac{1}{\lambda}\left|\dfrac{dN}{dt}\right| = \dfrac{7.56\times10^{11}\text{ decay/s}}{3.75\times10^{-4}\text{ s}^{-1}} = 2.02\times10^{15}$ nuclei.

**(b)** The number of nuclei left after one half-life is $\dfrac{N_0}{2} = 1.01\times10^{15}$ nuclei, and the activity is half:

$\left|\dfrac{dN}{dt}\right| = 3.78\times10^{11}$ decays/s.

**(c)** After three half-lives (92.4 minutes) there is an eighth of the original amount, so $N = 2.53\times10^{14}$ nuclei, and an eighth of the activity: $\left|\dfrac{dN}{dt}\right| = 9.45\times10^{10}$ decays/s.

**EVALUATE:** Since the activity is proportional to the number of radioactive nuclei that are present, the activity is halved in one half-life.

**43.30.** **IDENTIFY:** Apply $A = A_0 e^{-\lambda t}$.

**SET UP:** From Example 43.9, $\lambda = 1.21\times10^{-4}\text{ y}^{-1}$.

**EXECUTE:** The activity of the sample is $\dfrac{3070\text{ decays/min}}{(60\text{ sec/min})(0.500\text{ kg})} = 102$ Bq/kg, while the activity of atmospheric carbon is 255 Bq/kg (see Example 43.9). The age of the sample is then

$$t = -\frac{\ln(102/225)}{\lambda} = -\frac{\ln(102/225)}{1.21\times10^{-4}/\text{y}} = 7573\text{ y.}$$

**EVALUATE:** For $^{14}\text{C}$, $T_{1/2} = 5730$ y. The age is more than one half-life and the activity per kg of carbon is less than half the value when the tree died.

**43.31.** **IDENTIFY:** Knowing the equivalent dose in Sv, we want to find the absorbed energy.

**SET UP:** equivalent dose (Sv, rem) = RBE × absorbed dose(Gy, rad); 100 rad = 1 Gy

**EXECUTE:** **(a)** RBE = 1, so 0.25 mSv corresponds to 0.25 mGy.

Energy $= (0.25\times10^{-3}\text{ J/kg})/(5.0\text{ kg}) = 1.2\times10^{-3}$ J.

**(b)** RBE = 1 so 0.10 mGy = 10 mrad and 10 mrem. $(0.10\times10^{-3}\text{ J/kg})(75\text{ kg}) = 7.5\times10^{-3}$ J.

**EVALUATE:** **(c)** $\dfrac{7.5\times10^{-3}\text{ J}}{1.2\times10^{-3}\text{ J}} = 6.2.$ Each chest x ray delivers only about 1/6 of the yearly background radiation energy.

**43.32.** **IDENTIFY** and **SET UP:** The unit for absorbed dose is $1\text{ rad} = 0.01\text{ J/kg} = 0.01\text{ Gy}$. Equivalent dose in rem is RBE times absorbed dose in rad.

**EXECUTE:** **(a)** $\text{rem} = \text{rad} \times \text{RBE}$. $200 = x(10)$ and $x = 20$ rad.

**(b)** 1 rad deposits 0.010 J/kg, so 20 rad deposit 0.20 J/kg. This radiation affects 25 g (0.025 kg) of tissue, so the total energy is $(0.025\text{ kg})(0.20\text{ J/kg}) = 5.0 \times 10^{-3}\text{ J} = 5.0\text{ mJ}$.

**(c)** $\text{RBE} = 1$ for $\beta$-rays, so rem = rad. Therefore 20 rad = 20 rem.

**EVALUATE:** The same absorbed dose produces a larger equivalent dose when the radiation is neutrons than when it is electrons.

**43.33.** **IDENTIFY** and **SET UP:** The unit for absorbed dose is $1\text{ rad} = 0.01\text{ J/kg} = 0.01\text{ Gy}$. Equivalent dose in rem is RBE times absorbed dose in rad.

**EXECUTE:** $1\text{ rad} = 10^{-2}\text{ Gy}$, so $1\text{ Gy} = 100\text{ rad}$ and the dose was 500 rad.

$\text{rem} = (\text{rad})(\text{RBE}) = (500\text{ rad})(4.0) = 2000\text{ rem}$. $1\text{ Gy} = 1\text{ J/kg}$, so 5.0 J/kg.

**EVALUATE:** Gy, rad and J/kg are all units of absorbed dose. Rem is a unit of equivalent dose, which depends on the RBE of the radiation.

**43.34.** **IDENTIFY** and **SET UP:** For x rays $\text{RBE} = 1$ so the equivalent dose in Sv is the same as the absorbed dose in J/kg.

**EXECUTE:** One whole-body scan delivers $(75\text{ kg})(12 \times 10^{-3}\text{ J/kg}) = 0.90\text{ J}$. One chest x ray delivers $(5.0\text{ kg})(0.20 \times 10^{-3}\text{ J/kg}) = 1.0 \times 10^{-3}\text{ J}$. It takes $\dfrac{0.90\text{ J}}{1.0 \times 10^{-3}\text{ J}} = 900$ chest x rays to deliver the same total energy.

**EVALUATE:** For the CT scan the equivalent dose is much larger, and it is applied to the whole body.

**43.35.** **IDENTIFY** and **SET UP:** For x rays $\text{RBE} = 1$ and the equivalent dose equals the absorbed dose.

**EXECUTE:** **(a)** $175\text{ krad} = 175\text{ krem} = 1.75\text{ kGy} = 1.75\text{ kSv}$. $(1.75 \times 10^{3}\text{ J/kg})(0.220\text{ kg}) = 385\text{ J}$.

**(b)** $175\text{ krad} = 1.75\text{ kGy}$; $(1.50)(175\text{ krad}) = 262.5\text{ krem} = 2.625\text{ kSv}$. The energy deposited would be 385 J, the same as in (a).

**EVALUATE:** The energy required to raise the temperature of 0.150 kg of water 1 C° is 628 J, and 385 J is less than this. The energy deposited corresponds to a very small amount of heating.

**43.36.** **IDENTIFY:** $1\text{ rem} = 0.01\text{ Sv}$. Equivalent dose in rem equals RBE times the absorbed dose in rad. $1\text{ rad} = 0.01\text{ J/kg}$. To change the temperature of water, $Q = mc\Delta T$.

**SET UP:** For water, $c = 4190\text{ J/kg} \cdot \text{K}$.

**EXECUTE:** **(a)** $5.4\text{ Sv}(100\text{ rem/sv}) = 540\text{ rem}$.

**(b)** The RBE of 1 gives an absorbed dose of 540 rad.

**(c)** The absorbed dose is 5.4 Gy, so the total energy absorbed is $(5.4\text{ Gy})(65\text{ kg}) = 351\text{ J}$. The energy required to raise the temperature of 65 kg by 0.010° C is $(65\text{ kg})(4190\text{ J/kg} \cdot \text{K})(0.01\text{ C°}) = 3\text{ kJ}$.

**EVALUATE:** The amount of energy received corresponds to a very small heating of his body.

**43.37.** **IDENTIFY:** Apply Eq. (43.16), with $\lambda = \ln 2/T_{1/2}$, to find the number of tritium atoms that were ingested. Then use Eq. (43.17) to find the number of decays in one week.

**SET UP:** $1\text{ rad} = 0.01\text{ J/kg}$. $\text{rem} = \text{RBE} \times \text{rad}$.

**EXECUTE:** **(a)** We need to know how many decays per second occur.

$\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{(12.3\text{ y})(3.156 \times 10^{7}\text{ s/y})} = 1.785 \times 10^{-9}\text{ s}^{-1}$. The number of tritium atoms is

$N_0 = \dfrac{1}{\lambda}\left|\dfrac{dN}{dt}\right| = \dfrac{(0.35\text{ Ci})(3.70 \times 10^{10}\text{ Bq/Ci})}{1.79 \times 10^{-9}\text{ s}^{-1}} = 7.2540 \times 10^{18}$ nuclei. The number of remaining nuclei after one week is $N = N_0 e^{-\lambda t} = (7.25 \times 10^{18}) e^{-(1.79 \times 10^{-9}\text{ s}^{-1})(7)(24)(3600\text{ s})} = 7.2462 \times 10^{18}$ nuclei.

$\Delta N = N_0 - N = 7.8 \times 10^{15}$ decays. So the energy absorbed is

$E_{\text{total}} = \Delta N\ E_\gamma = (7.8\times10^{15})(5000\text{ eV})(1.60\times10^{-19}\text{ J/eV}) = 6.25\text{ J}.$ The absorbed dose is $\dfrac{6.25\text{ J}}{67\text{ kg}} = 0.0932\text{ J/kg} = 9.32\text{ rad}.$ Since $\text{RBE} = 1,$ then the equivalent dose is $9.32\text{ rem}.$

**EVALUATE: (b)** In the decay, antineutrinos are also emitted. These are not absorbed by the body, and so some of the energy of the decay is lost.

**43.38. IDENTIFY:** Each photon delivers energy. The energy of a single photon depends on its wavelength.

**SET UP:** equivalent dose (rem) = RBE × absorbed dose (rad). $1\text{ rad} = 0.010\text{ J/kg}.$ For x rays, $\text{RBE} = 1.$ Each photon has energy $E = \dfrac{hc}{\lambda}.$

**EXECUTE: (a)** $E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.0200\times10^{-9}\text{ m}} = 9.94\times10^{-15}\text{ J}.$ The absorbed energy is $(5.00\times10^{10}\text{ photons})(9.94\times10^{-15}\text{ J/photon}) = 4.97\times10^{-4}\text{ J} = 0.497\text{ mJ}.$

**(b)** The absorbed dose is $\dfrac{4.97\times10^{-4}\text{ J}}{0.600\text{ kg}} = 8.28\times10^{-4}\text{ J/kg} = 0.0828\text{ rad}.$ Since $\text{RBE} = 1,$ the equivalent dose is 0.0828 rem.

**EVALUATE:** The amount of energy absorbed is rather small (only ½ mJ), but it is absorbed by only 600 g of tissue.

**43.39. (a) IDENTIFY and SET UP:** Determine X by balancing the charge and nucleon number on the two sides of the reaction equation.

**EXECUTE:** X must have $A = 2 + 14 - 10 = 6$ and $Z = 1 + 7 - 5 = 3.$ Thus X is ${}^{6}_{3}\text{Li}$ and the reaction is

${}^{2}_{1}\text{H} + {}^{14}_{7}\text{N} \to {}^{6}_{3}\text{Li} + {}^{10}_{5}\text{B}.$

**(b) IDENTIFY and SET UP:** Calculate the mass decrease and find its energy equivalent.

**EXECUTE:** The neutral atoms on each side of the reaction equation have a total of 8 electrons, so the electron masses cancel when neutral atom masses are used. The neutral atom masses are found in Table 43.2.

mass of ${}^{2}_{1}\text{H} + {}^{14}_{7}\text{N}$ is $2.014102\text{ u} + 14.003074\text{ u} = 16.017176\text{ u}$

mass of ${}^{6}_{3}\text{Li} + {}^{10}_{5}\text{B}$ is $6.015121\text{ u} + 10.012937\text{ u} = 16.028058\text{ u}$

The mass increases, so energy is absorbed by the reaction. The $Q$ value is

$(16.017176\text{ u} - 16.028058\text{ u})(931.5\text{ MeV/u}) = -10.14\text{ MeV}$

**(c) IDENTIFY and SET UP:** The available energy in the collision, the kinetic energy $K_{\text{cm}}$ in the center of mass reference frame, is related to the kinetic energy $K$ of the bombarding particle by Eq. (43.24).

**EXECUTE:** The kinetic energy that must be available to cause the reaction is 10.14 MeV. Thus $K_{\text{cm}} = 10.14\text{ MeV}.$ The mass $M$ of the stationary target $({}^{14}_{7}\text{N})$ is $M = 14\text{ u}.$ The mass $m$ of the colliding particle $({}^{2}_{1}\text{H})$ is 2 u. Then by Eq. (43.24) the minimum kinetic energy $K$ that the ${}^{2}_{1}\text{H}$ must have is

$$K = \left(\frac{M+m}{M}\right)K_{\text{cm}} = \left(\frac{14\text{ u} + 2\text{ u}}{14\text{ u}}\right)(10.14\text{ MeV}) = 11.59\text{ MeV}.$$

**EVALUATE:** The projectile $({}^{2}_{1}\text{H})$ is much lighter than the target $({}^{14}_{7}\text{N})$ so $K$ is not much larger than $K_{\text{cm}}$. The $K$ we have calculated is what is required to allow the mass increase. We would also need to check to see if at this energy the projectile can overcome the Coulomb repulsion to get sufficiently close to the target nucleus for the reaction to occur.

**43.40. IDENTIFY:** The energy released is the energy equivalent of the mass decrease that occurs in the reaction.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** $m_{{}^{3}_{2}\text{He}} + m_{{}^{2}_{1}\text{H}} - m_{{}^{4}_{2}\text{He}} - m_{{}^{1}_{1}\text{H}} = 1.97\times10^{-2}\text{ u},$ so the energy released is $18.4\text{ MeV}.$

**EVALUATE:** Using neutral atom masses includes three electron masses on each side of the reaction equation and the same result is obtained as if nuclear masses had been used.

**43.41.** **IDENTIFY** and **SET UP:** Determine X by balancing the charge and the nucleon number on the two sides of the reaction equation.

**EXECUTE:** X must have $A = +2 + 9 - 4 = 7$ and $Z = +1 + 4 - 2 = 3$. Thus X is ${}^{7}_{3}\text{Li}$ and the reaction is

${}^{2}_{1}\text{H} + {}^{9}_{4}\text{Be} = {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$

**(b) IDENTIFY** and **SET UP:** Calculate the mass decrease and find its energy equivalent.

**EXECUTE:** If we use the neutral atom masses then there are the same number of electrons (five) in the reactants as in the products. Their masses cancel, so we get the same mass defect whether we use nuclear masses or neutral atom masses. The neutral atoms masses are given in Table 43.2.

${}^{2}_{1}\text{H} + {}^{9}_{4}\text{Be}$ has mass $2.014102\text{ u} + 9.012182\text{ u} = 11.26284\text{ u}$

${}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$ has mass $7.016003\text{ u} + 4.002603\text{ u} = 11.018606\text{ u}$

The mass decrease is $11.026284\text{ u} - 11.018606\text{ u} = 0.007678\text{ u}$.

This corresponds to an energy release of $0.007678\text{ u}(931.5\text{ MeV/1 u}) = 7.152\text{ MeV}$.

**(c) IDENTIFY** and **SET UP:** Estimate the threshold energy by calculating the Coulomb potential energy when the ${}^{2}_{1}\text{H}$ and ${}^{9}_{4}\text{Be}$ nuclei just touch. Obtain the nuclear radii from Eq. (43.1).

**EXECUTE:** The radius $R_{\text{Be}}$ of the ${}^{9}_{4}\text{Be}$ nucleus is $R_{\text{Be}} = (1.2\times10^{-15}\text{ m})(9)^{1/3} = 2.5\times10^{-15}\text{ m}$.

The radius $R_{\text{H}}$ of the ${}^{2}_{1}\text{H}$ nucleus is $R_{\text{H}} = (1.2\times10^{-15}\text{ m})(2)^{1/3} = 1.5\times10^{-15}\text{ m}$.

The nuclei touch when their center-to-center separation is

$R = R_{\text{Be}} + R_{\text{H}} = 4.0\times10^{-15}\text{ m}$.

The Coulomb potential energy of the two reactant nuclei at this separation is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e(4e)}{r}$$

$$U = (8.988\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.602\times10^{-19}\text{ C})^2}{(4.0\times10^{-15}\text{ m})(1.602\times10^{-19}\text{ J/eV})} = 1.4\text{ MeV}$$

This is an estimate of the threshold energy for this reaction.

**EVALUATE:** The reaction releases energy but the total initial kinetic energy of the reactants must be 1.4 MeV in order for the reacting nuclei to get close enough to each other for the reaction to occur. The nuclear force is strong but is very short-range.

**43.42.** **IDENTIFY** and **SET UP:** 0.7% of naturally occurring uranium is the isotope ${}^{235}\text{U}$. The mass of one ${}^{235}\text{U}$ nucleus is about $235m_{\text{p}}$.

**EXECUTE:** **(a)** The number of fissions needed is $\dfrac{1.0\times10^{19}\text{ J}}{(200\times10^{6}\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 3.13\times10^{29}$. The mass of ${}^{235}\text{U}$ required is $(3.13\times10^{29})(235m_{\text{p}}) = 1.23\times10^{5}\text{ kg}$.

**(b)** $\dfrac{1.23\times10^{5}\text{ kg}}{0.7\times10^{-2}} = 1.76\times10^{7}\text{ kg}$

**EVALUATE:** The calculation assumes 100% conversion of fission energy to electrical energy.

**43.43.** **IDENTIFY** and **SET UP:** The energy released is the energy equivalent of the mass decrease. 1 u is equivalent to 931.5 MeV. The mass of one ${}^{235}\text{U}$ nucleus is $235m_{\text{p}}$.

**EXECUTE:** **(a)** ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^{1}_{0}\text{n}$. We can use atomic masses since the same number of electrons are included on each side of the reaction equation and the electron masses cancel. The mass decrease is $\Delta M = m({}^{235}_{92}\text{U}) + m({}^{1}_{0}\text{n}) - [m({}^{144}_{56}\text{Ba}) + m({}^{89}_{36}\text{Kr}) + 3m({}^{1}_{0}\text{n})]$,

$\Delta M = 235.043930\text{ u} + 1.0086649\text{ u} - 143.922953\text{ u} - 88.917630\text{ u} - 3(1.0086649\text{ u})$, $\Delta M = 0.1860\text{ u}$. The energy released is $(0.1860\text{ u})(931.5\text{ MeV/u}) = 173.3\text{ MeV}$.

**(b)** The number of $^{235}\text{U}$ nuclei in 1.00 g is $\dfrac{1.00\times10^{-3}\text{ kg}}{235m_\text{p}}=2.55\times10^{21}$. The energy released per gram is

$(173.3\text{ MeV/nucleus})(2.55\times10^{21}\text{ nuclei/g})=4.42\times10^{23}\text{ MeV/g}$.

**EVALUATE:** The energy released is $7.1\times10^{10}$ J/kg. This is much larger than typical heats of combustion, which are about $5\times10^{4}$ J/kg.

**43.44.** **IDENTIFY:** The charge and the nucleon number are conserved. The energy of the photon must be at least as large as the energy equivalent of the mass increase in the reaction.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** **(a)** ${}^{28}_{14}\text{Si}+\gamma\Rightarrow{}^{24}_{12}\text{Mg}+{}^{A}_{Z}\text{X}$. $A+24=28$ so $A=4$. $Z+12=14$ so $Z=2$. X is an $\alpha$ particle.

**(b)** $-\Delta m=m({}^{24}_{12}\text{Mg})+m({}^{4}_{2}\text{He})-m({}^{28}_{14}\text{Si})=23.985042\text{ u}+4.002603\text{ u}-27.976927\text{ u}=0.010718\text{ u}$.

$E_\gamma=(-\Delta m)c^2=(0.010718\text{ u})(931.5\text{ MeV/u})=9.984\text{ MeV}$.

**EVALUATE:** The wavelength of the photon is

$\lambda=\dfrac{hc}{E}=\dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{9.984\times10^{6}\text{ eV}}=1.24\times10^{-13}\text{ m}=1.24\times10^{-4}\text{ nm}$. This is a gamma ray photon.

**43.45.** **IDENTIFY:** The energy released is the energy equivalent of the mass decrease that occurs in the reaction.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** The energy liberated will be

$M({}^{3}_{2}\text{He})+M({}^{4}_{2}\text{He})-M({}^{7}_{4}\text{Be})=(3.016029\text{ u}+4.002603\text{ u}-7.016929\text{ u})(931.5\text{ MeV/u})=1.586\text{ MeV}$.

**EVALUATE:** Using neutral atom masses includes four electrons on each side of the reaction equation and the result is the same as if nuclear masses had been used.

**43.46.** **IDENTIFY:** Charge and the number of nucleons are conserved in the reaction. The energy absorbed or released is determined by the mass change in the reaction.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** **(a)** $Z=3+2-0=5$ and $A=4+7-1=10$.

**(b)** The nuclide is a boron nucleus, and $m_\text{He}+m_\text{Li}-m_\text{n}-m_\text{B}=-3.00\times10^{-3}$ u, and so 2.79 MeV of energy is absorbed.

**EVALUATE:** The absorbed energy must come from the initial kinetic energy of the reactants.

**43.47.** **IDENTIFY:** First find the number of deuterium nuclei in the water. Each fusion event requires two of them, and each such event releases 4.03 MeV of energy.

**SET UP** and **EXECUTE:** The molecular mass of water is $18.015\times10^{-3}$ kg/mol. $m=\rho V$ so the 100.0 $\text{cm}^3$ sample has a mass of $m=(1000\text{ kg/m}^3)(100.0\times10^{-6}\text{ m}^3)=0.100$ kg. The sample contains 5.551 moles and $(5.551\text{ mol})(6.022\times10^{23}\text{ molecules/mol})=3.343\times10^{24}$ molecules. The number of $D_2O$ molecules is $5.014\times10^{20}$. Each molecule contains the two deuterons needed for one fusion reaction. Therefore, the energy liberated is $(5.014\times10^{20})(4.03\times10^{6}\text{ eV})=2.021\times10^{27}\text{ eV}=3.24\times10^{8}$ J.

**EVALUATE:** This is about 300 million joules of energy! And after the fusion, essentially the same amount of water would remain since it is only the tiny percent that is deuterium that undergoes fusion.

**43.48.** **IDENTIFY** and **SET UP:** $m=\rho V$. $1\text{ gal}=3.788\text{ L}=3.788\times10^{-3}\text{ m}^3$. The mass of a $^{235}\text{U}$ nucleus is $235m_\text{p}$. $1\text{ MeV}=1.60\times10^{-13}$ J

**EXECUTE:** **(a)** For 1 gallon, $m=\rho V=(737\text{ kg/m}^3)(3.788\times10^{-3}\text{ m}^3)=2.79\text{ kg}=2.79\times10^{3}$ g

$\dfrac{1.3\times10^{8}\text{ J/gal}}{2.79\times10^{3}\text{ g/gal}}=4.7\times10^{4}\text{ J/g}$

**(b)** 1 g contains $\dfrac{1.00\times10^{-3}\text{ kg}}{235m_\text{p}} = 2.55\times10^{21}$ nuclei

$(200\text{ MeV/nucleus})(1.60\times10^{-13}\text{ J/MeV})(2.55\times10^{21}\text{ nuclei}) = 8.2\times10^{10}\text{ J/g}$

**(c)** A mass of $6m_\text{p}$ produces 26.7 MeV.

$$\frac{(26.7\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{6m_\text{p}} = 4.26\times10^{14}\text{ J/kg} = 4.26\times10^{11}\text{ J/g}$$

**(d)** The total energy available would be $(1.99\times10^{30}\text{ kg})(4.7\times10^{7}\text{ J/kg}) = 9.4\times10^{37}\text{ J}$

$$\text{power} = \frac{\text{energy}}{t} \text{ so } t = \frac{\text{energy}}{\text{power}} = \frac{9.4\times10^{37}\text{ J}}{3.86\times10^{26}\text{ W}} = 2.4\times10^{11}\text{ s} = 7600\text{ yr}$$

**EVALUATE:** If the mass of the sun were all proton fuel, it would contain enough fuel to last $(7600\text{ yr})\left(\dfrac{4.3\times10^{11}\text{ J/g}}{4.7\times10^{4}\text{ J/g}}\right) = 7.0\times10^{10}$ yr.

**43.49.** **IDENTIFY** and **SET UP:** Follow the procedure specified in the hint.

**EXECUTE:** Nuclei: ${}^{A}_{Z}\text{X}^{Z+} \to {}^{A-4}_{Z-2}\text{Y}^{(Z-2)+} + {}^{4}_{2}\text{He}^{2+}$. Add the mass of $Z$ electrons to each side and we find: $\Delta m = M({}^{A}_{Z}\text{X}) - M({}^{A-4}_{Z-2}\text{Y}) - M({}^{4}_{2}\text{He})$, where now we have the mass of the neutral atoms. So as long as the mass of the original neutral atom is greater than the sum of the neutral products masses, the decay can happen.

**EVALUATE:** The energy released in the decay is the energy equivalent of $\Delta m$.

**43.50.** **IDENTIFY** and **SET UP:** Follow the procedure specified in the hint in Problem 43.49.

**EXECUTE:** Denote the reaction as ${}^{A}_{Z}\text{X} \to {}_{Z+1}{}^{A}\text{Y} + \text{e}^-$. The mass defect is related to the change in the neutral atomic masses by $[m_\text{X} - Zm_\text{e}] - [m_\text{Y} - (Z+1)m_\text{e}] - m_\text{e} = (m_\text{X} - m_\text{Y})$, where $m_\text{X}$ and $m_\text{Y}$ are the masses as tabulated in, for instance, Table (43.2).

**EVALUATE:** It is essential to correctly account for the electron masses.

**43.51.** **IDENTIFY** and **SET UP:** Follow the procedure specified in the hint in Problem 43.49.

**EXECUTE:** ${}^{A}_{Z}\text{X}^{Z+} \to {}_{Z-1}{}^{A}\text{Y}^{(Z-1)+} + \beta^+$. Adding $(Z-1)$ electrons to both sides yields ${}^{A}_{Z}\text{X}^+ \to {}_{Z-1}{}^{A}\text{Y} + \beta^+$.

So in terms of masses:

$$\Delta m = M({}^{A}_{Z}\text{X}^+) - M({}_{Z-1}{}^{A}\text{Y}) - m_\text{e} = (M({}^{A}_{Z}\text{X}) - m_\text{e}) - M({}_{Z-1}{}^{A}\text{Y}) - m_\text{e} = M({}^{A}_{Z}\text{X}) - M({}_{Z-1}{}^{A}\text{Y}) - 2m_\text{e}.$$

So the decay will occur as long as the original neutral mass is greater than the sum of the neutral product mass and two electron masses.

**EVALUATE:** It is essential to correctly account for the electron masses.

**43.52.** **IDENTIFY:** The minimum energy to remove a proton from the nucleus is equal to the energy difference between the two states of the nucleus (before and after proton removal).

**(a) SET UP:** ${}^{12}_{6}\text{C} = {}^{1}_{1}\text{H} + {}^{11}_{5}\text{B}$. $\Delta m = m({}^{1}_{1}\text{H}) + m({}^{11}_{5}\text{B}) - m({}^{12}_{6}\text{C})$. The electron masses cancel when neutral atom masses are used.

**EXECUTE:** $\Delta m = 1.007825\text{ u} + 11.009305\text{ u} - 12.000000\text{ u} = 0.01713\text{ u}$. The energy equivalent of this mass increase is $(0.01713\text{ u})(931.5\text{ MeV/u}) = 16.0\text{ MeV}$.

**(b) SET UP** and **EXECUTE:** We follow the same procedure as in part (a).

$$\Delta M = 6M_\text{H} + 6M_\text{n} - {}^{12}_{6}M = 6(1.007825\text{ u}) + 6(1.008665\text{ u}) - 12.000000\text{ u} = 0.09894\text{ u}.$$

$$E_\text{B} = (0.09894\text{ u})(931.5\text{ MeV/u}) = 92.16\text{ MeV}. \quad \frac{E_\text{B}}{A} = 7.68\text{ MeV/u}.$$

**EVALUATE:** The proton removal energy is about twice the binding energy per nucleon.

**43.53.** **IDENTIFY:** The minimum energy to remove a proton or a neutron from the nucleus is equal to the energy difference between the two states of the nucleus, before and after removal.

**(a) SET UP:** ${}^{17}_{8}\text{O} = {}^{1}_{0}\text{n} + {}^{16}_{8}\text{O}$. $\Delta m = m({}^{1}_{0}\text{n}) + m({}^{16}_{8}\text{O}) - m({}^{17}_{8}\text{O})$. The electron masses cancel when neutral atom masses are used.

**EXECUTE:** $\Delta m = 1.008665\text{ u} + 15.994915\text{ u} - 16.999132\text{ u} = 0.004448\text{ u}$. The energy equivalent of this mass increase is $(0.004448\text{ u})(931.5\text{ MeV/u}) = 4.14\text{ MeV}$.

**(b) SET UP and EXECUTE:** Following the same procedure as in part (a) gives

$\Delta M = 8M_{\text{H}} + 9M_{\text{n}} - {}^{17}_{8}M = 8(1.007825\text{ u}) + 9(1.008665\text{ u}) - 16.999132\text{ u} = 0.1415\text{ u}$.

$E_{\text{B}} = (0.1415\text{ u})(931.5\text{ MeV/u}) = 131.8\text{ MeV}$. $\dfrac{E_{\text{B}}}{A} = 7.75\text{ MeV/nucleon}$.

**EVALUATE:** The neutron removal energy is about half the binding energy per nucleon.

**43.54.** **IDENTIFY:** The minimum energy to remove a proton or a neutron from the nucleus is equal to the energy difference between the two states of the nucleus, before and after removal.

**SET UP and EXECUTE:** proton removal: ${}^{15}_{7}\text{N} = {}^{1}_{1}\text{H} + {}^{14}_{6}\text{C}$, $\Delta m = m({}^{1}_{1}\text{H}) + m({}^{14}_{6}\text{C}) - m({}^{15}_{7}\text{N})$. The electron masses cancel when neutral atom masses are used.

$\Delta m = 1.007825\text{ u} + 14.003242\text{ u} - 15.000109\text{ u} = 0.01096\text{ u}$. The proton removal energy is 10.2 MeV.

neutron removal: ${}^{15}_{7}\text{N} = {}^{1}_{0}\text{n} + {}^{14}_{7}\text{N}$. $\Delta m = m({}^{1}_{0}\text{n}) + m({}^{14}_{7}\text{N}) - m({}^{15}_{7}\text{N})$. The electron masses cancel when neutral atom masses are used.

$\Delta m = 1.008665\text{ u} + 14.003074\text{ u} - 15.000109\text{ u} = 0.01163\text{ u}$. The neutron removal energy is 10.8 MeV.

**EVALUATE:** The neutron removal energy is 6% larger than the proton removal energy.

**43.55.** **IDENTIFY:** Use the decay scheme and half-life of ${}^{90}\text{Sr}$ to find out the product of its decay and the amount left after a given time.

**SET UP:** The particle emitted in $\beta^-$ decay is an electron, ${}^{0}_{-1}\text{e}$. In a time of one half-life, the number of radioactive nuclei decreases by a factor of 2. $6.25\% = \dfrac{1}{16} = 2^{-4}$

**EXECUTE:** **(a)** ${}^{90}_{38}\text{Sr} \rightarrow {}^{0}_{-1}\text{e} + {}^{90}_{39}\text{Y}$. The daughter nucleus is ${}^{90}_{39}\text{Y}$.

**(b)** 56 y is $2T_{1/2}$ so $N = N_0/2^2 = N_0/4$; 25% is left.

**(c)** $\dfrac{N}{N_0} = 2^{-n}$; $\dfrac{N}{N_0} = 6.25\% = \dfrac{1}{16} = 2^{-4}$ so $t = 4T_{1/2} = 112\text{ y}$.

**EVALUATE:** After half a century, ¼ of the ${}^{90}\text{Sr}$ would still be left!

**43.56.** **IDENTIFY:** Calculate the mass defect for the decay. Example 43.5 uses conservation of linear momentum to determine how the released energy is divided between the decay partners.

**SET UP:** 1 u is equivalent to 931.5 MeV.

**EXECUTE:** The $\alpha$-particle will have $\dfrac{226}{230}$ of the released energy (see Example 43.5).

$\dfrac{226}{230}(m_{\text{Th}} - m_{\text{Ra}} - m_{\alpha}) = 5.032\times10^{-3}\text{ u or }4.69\text{ MeV}$.

**EVALUATE:** Most of the released energy goes to the $\alpha$ particle, since its mass is much less than that of the daughter nucleus.

**43.57.** **(a) IDENTIFY and SET UP:** The heavier nucleus will decay into the lighter one.

**EXECUTE:** ${}^{25}_{13}\text{Al}$ will decay into ${}^{25}_{12}\text{Mg}$.

**(b) IDENTIFY and SET UP:** Determine the emitted particle by balancing $A$ and $Z$ in the decay reaction.

**EXECUTE:** This gives ${}^{25}_{13}\text{Al} \rightarrow {}^{25}_{12}\text{Mg} + {}^{0}_{+1}\text{e}$. The emitted particle must have charge $+e$ and its nucleon number must be zero. Therefore, it is a $\beta^+$ particle, a positron.

**(c) IDENTIFY and SET UP:** Calculate the energy defect $\Delta M$ for the reaction and find the energy equivalent of $\Delta M$. Use the nuclear masses for ${}^{25}_{13}\text{Al}$ and ${}^{25}_{12}\text{Mg}$, to avoid confusion in including the correct number of electrons if neutral atom masses are used.

**EXECUTE:** The nuclear mass for ${}^{25}_{13}\text{Al}$ is $M_{\text{nuc}}({}^{25}_{13}\text{Al}) = 24.990429\text{ u} - 13(0.000548580\text{ u}) = 24.983297\text{ u}$.

The nuclear mass for ${}^{25}_{12}\text{Mg}$ is $M_{\text{nuc}}({}^{25}_{12}\text{Mg}) = 24.985837\text{ u} - 12(0.000548580\text{ u}) = 24.979254\text{ u}$.

The mass defect for the reaction is

$\Delta M = M_{\text{nuc}}(^{25}_{13}\text{Al}) - M_{\text{nuc}}(^{25}_{12}\text{Mg}) - M(^{0}_{+1}\text{e}) = 24.983297\text{ u} - 24.979254\text{ u} - 0.00054858\text{ u} = 0.003494\text{ u}$

$Q = (\Delta M)c^2 = 0.003494\text{ u}(931.5\text{ MeV/1 u}) = 3.255\text{ MeV}$

**EVALUATE:** The mass decreases in the decay and energy is released. Note: $^{25}_{13}$Al can also decay into $^{25}_{12}$Mg by the electron capture.

$^{25}_{13}\text{Al} + {}^{0}_{-1}\text{e} \rightarrow {}^{25}_{12}\text{Mg}$

The $^{0}_{-1}$ electron in the reaction is an orbital electron in the neutral $^{25}_{13}$Al atom. The mass defect can be calculated using the nuclear masses:

$\Delta M = M_{\text{nuc}}(^{25}_{13}\text{Al}) + M(^{0}_{-1}\text{e}) - M_{\text{nuc}}(^{25}_{12}\text{Mg}) = 24.983287\text{ u} + 0.00054858\text{ u} - 24.979254\text{ u} = 0.004592\text{ u}.$

$Q = (\Delta M)c^2 = (0.004592\text{ u})(931.5\text{ MeV/1 u}) = 4.277\text{ MeV}$

The mass decreases in the decay and energy is released.

**43.58.** **IDENTIFY:** Calculate the mass change in the decay. If the mass decreases the decay is energetically allowed.

**SET UP:** Example 43.5 shows how the released energy is distributed among the decay products for $\alpha$ decay.

**EXECUTE:** **(a)** $m_{^{210}_{84}\text{Po}} - m_{^{206}_{82}\text{Pb}} - m_{^{4}_{2}\text{He}} = 5.81\times10^{-3}$ u, or $Q = 5.41$ MeV. The energy of the alpha particle is $(206/210)$ times this, or 5.30 MeV (see Example 43.5).

**(b)** $m_{^{210}_{84}\text{Po}} - m_{^{209}_{83}\text{Bi}} - m_{^{1}_{1}\text{H}} = -5.35\times10^{-3}\text{ u} < 0,$ so the decay is not possible.

**(c)** $m_{^{210}_{84}\text{Po}} - m_{^{209}_{84}\text{Po}} - m_{\text{n}} = -8.22\times10^{-3}\text{ u} < 0,$ so the decay is not possible.

**(d)** $m_{^{210}_{85}\text{At}} > m_{^{210}_{84}\text{Po}},$ so the decay is not possible (see Problem (43.50)).

**(e)** $m_{^{210}_{83}\text{Bi}} + 2m_{\text{e}} > m_{^{210}_{84}\text{Po}},$ so the decay is not possible (see Problem (43.51)).

**EVALUATE:** Of the decay processes considered in the problem, only $\alpha$ decay is energetically allowed for $^{210}_{84}$Po.

**43.59.** **IDENTIFY** and **SET UP:** The amount of kinetic energy released is the energy equivalent of the mass change in the decay. $m_{\text{e}} = 0.0005486$ u and the atomic mass of $^{14}_{7}$N is 14.003074 u. The energy equivalent of 1 u is 931.5 MeV. $^{14}$C has a half-life of $T_{1/2} = 5730\text{ yr} = 1.81\times10^{11}$ s. The RBE for an electron is 1.0.

**EXECUTE:** **(a)** $^{14}_{6}\text{C} \rightarrow \text{e}^- + {}^{14}_{7}\text{N} + \bar{\nu}_{\text{e}}.$

**(b)** The mass decrease is $\Delta M = m(^{14}_{6}\text{C}) - [m_{\text{e}} + m(^{14}_{7}\text{N})].$ Use nuclear masses, to avoid difficulty in accounting for atomic electrons. The nuclear mass of $^{14}_{6}$C is $14.003242\text{ u} - 6m_{\text{e}} = 13.999950$ u. The nuclear mass of $^{14}_{7}$N is $14.003074\text{ u} - 7m_{\text{e}} = 13.999234$ u.

$\Delta M = 13.999950\text{ u} - 13.999234\text{ u} - 0.000549\text{ u} = 1.67\times10^{-4}$ u. The energy equivalent of $\Delta M$ is 0.156 MeV.

**(c)** The mass of carbon is $(0.18)(75\text{ kg}) = 13.5$ kg. From Example 43.9, the activity due to 1 g of carbon in a living organism is 0.255 Bq. The number of decay/s due to 13.5 kg of carbon is $(13.5\times10^3\text{ g})(0.255\text{ Bq/g}) = 3.4\times10^3$ decays/s.

**(d)** Each decay releases 0.156 MeV so $3.4\times10^3$ decays/s releases $530\text{ MeV/s} = 8.5\times10^{-11}$ J/s.

**(e)** The total energy absorbed in 1 year is $(8.5\times10^{-11}\text{ J/s})(3.156\times10^7\text{ s}) = 2.7\times10^{-3}$ J. The absorbed dose is $\dfrac{2.7\times10^{-3}\text{ J}}{75\text{ kg}} = 3.6\times10^{-5}\text{ J/kg} = 36\ \mu\text{Gy} = 3.6$ mrad. With RBE = 1.0, the equivalent dose is $36\ \mu\text{Sv} = 3.6$ mrem.

**EVALUATE:** Section 43.5 says that background radiation exposure is about 1.0 mSv per year. The radiation dose calculated in this problem is much less than this.

**43.60.** **IDENTIFY and SET UP:** $m_\pi = 264m_e = 2.40\times10^{-28}$ kg. The total energy of the two photons equals the rest mass energy $m_\pi c^2$ of the pion.

**EXECUTE:** **(a)** $E_{ph} = \frac{1}{2}m_\pi c^2 = \frac{1}{2}(2.40\times10^{-28}\text{ kg})(3.00\times10^8\text{ m/s})^2 = 1.08\times10^{-11}\text{ J} = 67.5\text{ MeV}$

$E_{ph} = \dfrac{hc}{\lambda}$ so $\lambda = \dfrac{hc}{E_{ph}} = \dfrac{1.24\times10^{-6}\text{ eV}\cdot\text{m}}{67.5\times10^6\text{ eV}} = 1.84\times10^{-14}\text{ m} = 18.4\text{ fm}$

These are gamma ray photons, so they have RBE = 1.0.

**(b)** Each pion delivers $2(1.08\times10^{-11}\text{ J}) = 2.16\times10^{-11}$ J.

The absorbed dose is 200 rad = 2.00 Gy = 2.00 J/kg.

The energy deposited is $(25\times10^{-3}\text{ kg})(2.00\text{ J/kg}) = 0.050$ J.

The number of $\pi^0$ mesons needed is $\dfrac{0.050\text{ J}}{2.16\times10^{-11}\text{ J/meson}} = 2.3\times10^9$ mesons.

**EVALUATE:** Note that charge is conserved in the decay since the pion is neutral. If the pion is initially at rest the photons must have equal momenta in opposite directions so the two photons have the same $\lambda$ and are emitted in opposite directions. The photons also have equal energies since they have the same momentum and $E = pc$.

**43.61.** **IDENTIFY and SET UP:** Find the energy equivalent of the mass decrease. Part of the released energy appears as the emitted photon and the rest as kinetic energy of the electron.

**EXECUTE:** ${}^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e}$

The mass change is $197.968225\text{ u} - 197.966752\text{ u} = 1.473\times10^{-3}$ u

(The neutral atom masses include 79 electrons before the decay and 80 electrons after the decay. This one additional electron in the product accounts correctly for the electron emitted by the nucleus.) The total energy released in the decay is $(1.473\times10^{-3}\text{ u})(931.5\text{ MeV/u}) = 1.372$ MeV. This energy is divided between the energy of the emitted photon and the kinetic energy of the $\beta^-$ particle. Thus the $\beta^-$ particle has kinetic energy equal to $1.372\text{ MeV} - 0.412\text{ MeV} = 0.960$ MeV.

**EVALUATE:** The emitted electron is much lighter than the ${}^{198}_{80}\text{Hg}$ nucleus, so the electron has almost all the final kinetic energy. The final kinetic energy of the ${}^{198}\text{Hg}$ nucleus is very small.

**43.62.** **IDENTIFY and SET UP:** Problem 43.51 shows how to calculate the mass defect using neutral atom masses.

**EXECUTE:** $m_{{}^{11}_{6}\text{C}} - m_{{}^{11}_{5}\text{B}} - 2m_e = 1.03\times10^{-3}$ u. Decay is energetically possible.

**EVALUATE:** The energy released in the decay is $(1.03\times10^{-3}\text{ u})(931.5\text{ MeV/u}) = 0.959$ MeV.

**43.63.** **IDENTIFY and SET UP:** The decay is energetically possible if the total mass decreases. Determine the nucleus produced by the decay by balancing *A* and *Z* on both sides of the equation. ${}^{13}_{7}\text{N} \rightarrow {}^{0}_{+1}\text{e} + {}^{13}_{6}\text{C}$. To avoid confusion in including the correct number of electrons with neutral atom masses, use nuclear masses, obtained by subtracting the mass of the atomic electrons from the neutral atom masses.

**EXECUTE:** The nuclear mass for ${}^{13}_{7}\text{N}$ is $M_{nuc}({}^{13}_{7}\text{N}) = 13.005739\text{ u} - 7(0.00054858\text{ u}) = 13.001899$ u.

The nuclear mass for ${}^{13}_{6}\text{C}$ is $M_{nuc}({}^{13}_{6}\text{C}) = 13.003355\text{ u} - 6(0.00054858\text{ u}) = 13.000064$ u.

The mass defect for the reaction is

$\Delta M = M_{nuc}({}^{13}_{7}\text{N}) - M_{nuc}({}^{13}_{6}\text{C}) - M({}^{0}_{+1}\text{e})$. $\Delta M = 13.001899\text{ u} - 13.000064\text{ u} - 0.00054858\text{ u} = 0.001286$ u.

**EVALUATE:** The mass decreases in the decay, so energy is released. This decay is energetically possible.

**43.64.** **IDENTIFY:** Apply $\left|\dfrac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$, with $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

**SET UP:** $\ln|dN/dt| = \ln \lambda N_0 - \lambda t$

**EXECUTE:** **(a)** A least-squares fit to log of the activity *vs.* time gives a slope of magnitude $\lambda = 0.5995\ \text{h}^{-1}$, for a half-life of $\dfrac{\ln 2}{\lambda} = 1.16\ \text{h}$.

**(b)** The initial activity is $N_0\lambda$, and this gives $N_0 = \dfrac{(2.00\times10^4\ \text{Bq})}{(0.5995\ \text{hr}^{-1})(1\ \text{hr}/3600\ \text{s})} = 1.20\times10^8$.

**(c)** $N = N_0 e^{-\lambda t} = 1.81\times10^6$.

**EVALUATE:** The activity decreases by about $\frac{1}{2}$ in the first hour, so the half-life is about 1 hour.

**43.65.** **IDENTIFY:** Assume the activity is constant during the year and use the given value of the activity to find the number of decays that occur in one year. Absorbed dose is the energy absorbed per mass of tissue. Equivalent dose is RBE times absorbed dose.

**SET UP:** For $\alpha$ particles, $\text{RBE} = 20$ (from Table 43.3).

**EXECUTE:** $(0.63\times10^{-6}\ \text{Ci})(3.7\times10^{10}\ \text{Bq/Ci})(3.156\times10^7\ \text{s}) = 7.357\times10^{11}\ \alpha$ particles. The absorbed dose is $\dfrac{(7.357\times10^{11})(4.0\times10^6\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}{(0.50\ \text{kg})} = 0.943\ \text{Gy} = 94.3\ \text{rad}$. The equivalent dose is (20) $(94.3\ \text{rad}) = 1900\ \text{rem}$.

**EVALUATE:** The equivalent dose is 19 Sv. This is large enough for significant damage to the person.

**43.66.** **IDENTIFY** and **SET UP:** $T_{1/2} = \dfrac{\ln 2}{\lambda}$. The mass of a single nucleus is $149m_\text{p} = 2.49\times10^{-25}\ \text{kg}$. $dN/dt = -\lambda N$.

**EXECUTE:** $N = \dfrac{12.0\times10^{-3}\ \text{kg}}{2.49\times10^{-25}\ \text{kg}} = 4.82\times10^{22}$. $dN/dt = -2.65$ decays/s.

$$\lambda = -\frac{dN/dt}{N} = \frac{2.65\ \text{decays/s}}{4.82\times10^{22}} = 5.50\times10^{-23}\ \text{s}^{-1};\ T_{1/2} = \frac{\ln 2}{\lambda} = 1.26\times10^{22}\ \text{s} = 3.99\times10^{14}\ \text{y}.$$

**EVALUATE:** The half-life determines the fraction of nuclei in a sample that decay each second.

**43.67.** **IDENTIFY** and **SET UP:** One-half of the sample decays in a time of $T_{1/2}$.

**EXECUTE:** **(a)** $\dfrac{10\times10^9\ \text{y}}{200{,}000\ \text{y}} = 5.0\times10^4$.

**(b)** $\left(\frac{1}{2}\right)^{5.0\times10^4}$. This exponent is too large for most hand-held calculators. But $\left(\frac{1}{2}\right) = 10^{-0.301}$, so $\left(\frac{1}{2}\right)^{5.0\times10^4} = (10^{-0.301})^{5.0\times10^4} = 10^{-15{,}000}$.

**EVALUATE:** For $N = 1$ after 16 billion years, $N_0 = 10^{15{,}000}$. The mass of this many $^{99}\text{Tc}$ nuclei would be $(99)(1.66\times10^{-27}\ \text{kg})(10^{15{,}000}) = 10^{14{,}750}\ \text{kg}$, which is immense, far greater than the mass of any star.

**43.68.** **IDENTIFY:** One rad of absorbed dose is 0.01 J/kg. The equivalent dose in rem is the absorbed dose in rad times the RBE. For part (c) apply Eq. (43.16) with $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

**SET UP:** For $\alpha$ particles, $\text{RBE} = 20$ (Table 43.3).

**EXECUTE:** **(a)** $(6.25\times10^{12})(4.77\times10^6\ \text{MeV})(1.602\times10^{-19}\ \text{J/eV})/(70.0\ \text{kg}) = 0.0682\ \text{Gy} = 0.682\ \text{rad}$.

**(b)** $(20)(6.82\ \text{rad}) = 136\ \text{rem}$.

**(c)** $\left|\dfrac{dN}{dt}\right| = \dfrac{m}{Am_\text{p}}\dfrac{\ln(2)}{T_{1/2}} = 1.17\times10^9\ \text{Bq} = 31.6\ \text{mCi}$.

**(d)** $t = \dfrac{6.25\times10^{12}}{1.17\times10^9\ \text{Bq}} = 5.34\times10^3\ \text{s}$, about an hour and a half.

**EVALUATE:** The time in part (d) is so small in comparison with the half-life that the decrease in activity of the source may be neglected.

**43.69.** **IDENTIFY:** Use Eq. (43.17) to relate the initial number of radioactive nuclei, $N_0$, to the number, *N*, left after time *t*.

**SET UP:** We have to be careful; after $^{87}\text{Rb}$ has undergone radioactive decay it is no longer a rubidium atom. Let $N_{85}$ be the number of $^{85}\text{Rb}$ atoms; this number doesn't change. Let $N_0$ be the number of $^{87}\text{Rb}$ atoms on earth when the solar system was formed. Let *N* be the present number of $^{87}\text{Rb}$ atoms.

**EXECUTE:** The present measurements say that $0.2783 = N/(N + N_{85})$.

$(N + N_{85})(0.2783) = N$, so $N = 0.3856N_{85}$. The percentage we are asked to calculate is $N_0/(N_0 + N_{85})$.

$N$ and $N_0$ are related by $N = N_0 e^{-\lambda t}$ so $N_0 = e^{+\lambda t}N$.

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{Ne^{\lambda t}}{Ne^{\lambda t} + N_{85}} = \dfrac{(0.3855e^{\lambda t})N_{85}}{(0.3856e^{\lambda t})N_{85} + N_{85}} = \dfrac{0.3856e^{\lambda t}}{0.3856e^{\lambda t} + 1}$.

$t = 4.6\times10^9$ y; $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{4.75\times10^{10}\text{ y}} = 1.459\times10^{-11}\text{ y}^{-1}$

$e^{\lambda t} = e^{(1.459\times10^{-11}\text{ y}^{-1})(4.6\times10^9\text{ y})} = e^{0.06711} = 1.0694$

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{(0.3856)(1.0694)}{(0.3856)(1.0694) + 1} = 29.2\%$.

**EVALUATE:** The half-life for $^{87}\text{Rb}$ is a factor of 10 larger than the age of the solar system, so only a small fraction of the $^{87}\text{Rb}$ nuclei initially present have decayed; the percentage of rubidium atoms that are radioactive is only a bit less now than it was when the solar system was formed.

**43.70.** **IDENTIFY:** From Example 43.5, the kinetic energy of the particle is $K = \dfrac{M_\alpha}{M_\alpha + m}K_\infty$, where $K_\infty$ is the energy that the $\alpha$-particle would have if the nucleus were infinitely massive. $K_\infty$ is equal to the total energy released in the reaction. The energy released in the reaction is the energy equivalent of the mass decrease in the reaction.

**SET UP:** 1 u is equivalent to 931.5 MeV. The atomic mass of $^4_2\text{He}$ is 4.002603 u.

**EXECUTE:** $M = M_{\text{Os}} - M_\alpha - K_\infty = M_{\text{Os}} - M_\alpha - \dfrac{186}{182}(2.76\text{ MeV}/c^2) = 181.94821\text{ u}$.

**EVALUATE:** The daughter nucleus is $^{182}_{74}\text{W}$.

**43.71.** **IDENTIFY** and **SET UP:** Find the energy emitted and the energy absorbed each second. Convert the absorbed energy to absorbed dose and to equivalent dose.

**EXECUTE:** **(a)** First find the number of decays each second:

$2.6\times10^{-4}\text{ Ci}\left(\dfrac{3.70\times10^{10}\text{ decays/s}}{1\text{ Ci}}\right) = 9.6\times10^6$ decays/s. The average energy per decay is 1.25 MeV, and one-half of this energy is deposited in the tumor. The energy delivered to the tumor per second then is $\frac{1}{2}(9.6\times10^6\text{ decays/s})(1.25\times10^6\text{ eV/decay})(1.602\times10^{-19}\text{ J/eV}) = 9.6\times10^{-7}\text{ J/s}$.

**(b)** The absorbed dose is the energy absorbed divided by the mass of the tissue:

$\dfrac{9.6\times10^{-7}\text{ J/s}}{0.200\text{ kg}} = (4.8\times10^{-6}\text{ J/kg}\cdot\text{s})(1\text{ rad}/(0.01\text{ J/kg})) = 4.8\times10^{-4}\text{ rad/s}$.

**(c)** equivalent dose (REM) = RBE × absorbed dose (rad). In one second the equivalent dose is

$(0.70)(4.8\times10^{-4}\text{ rad}) = 3.4\times10^{-4}\text{ rem}$.

**(d)** $(200\text{ rem})/(3.4\times10^{-4}\text{ rem/s}) = (5.9\times10^5\text{ s})(1\text{ h}/3600\text{ s}) = 164\text{ h} = 6.9\text{ days}$.

**EVALUATE:** The activity of the source is small so that absorbed energy per second is small and it takes several days for an equivalent dose of 200 rem to be absorbed by the tumor. A 200-rem dose equals 2.00 Sv and this is large enough to damage the tissue of the tumor.

**43.72.** **IDENTIFY:** Apply Eq. (43.17), with $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

**SET UP:** Let 1 refer to ${}^{15}_{8}\text{O}$ and 2 refer to ${}^{19}_{8}\text{O}$. $\dfrac{N_1}{N_2} = \dfrac{e^{-\lambda_1 t}}{e^{-\lambda_2 t}}$, since $N_0$ is the same for the two isotopes.

$e^{-\lambda t} = e^{-(\ln 2/T_{1/2})t} = (e^{-\ln 2})^{t/T_{1/2}} = (\tfrac{1}{2})^{t/T_{1/2}}$. $\dfrac{N_1}{N_2} = \left(\dfrac{1}{2}\right)^{(t/(T_{1/2})_1)/(t/(T_{1/2})_2)} = 2^{t\left(\frac{1}{(T_{1/2})_2} - \frac{1}{(T_{1/2})_1}\right)}$.

**EXECUTE:** **(a)** After $4.0 \text{ min} = 240 \text{ s}$, the ratio of the number of nuclei is

$$\frac{2^{-240/122.2}}{2^{-240/26.9}} = 2^{(240)\left(\frac{1}{26.9} - \frac{1}{122.2}\right)} = 124.$$

**(b)** After $15.0 \text{ min} = 900 \text{ s}$, the ratio is $7.15 \times 10^7$.

**EVALUATE:** The ${}^{19}_{8}\text{O}$ nuclei decay at a greater rate, so the ratio $N({}^{15}_{8}\text{O})/N({}^{19}_{8}\text{O})$ increases with time.

**43.73.** **IDENTIFY** and **SET UP:** The number of radioactive nuclei left after time *t* is given by $N = N_0 e^{-\lambda t}$. The problem says $N/N_0 = 0.29$; solve for *t*.

**EXECUTE:** $0.29 = e^{-\lambda t}$ so $\ln(0.29) = -\lambda t$ and $t = -\ln(0.29)/\lambda$. Example 43.9 gives $\lambda = 1.209 \times 10^{-4} \text{ y}^{-1}$ for ${}^{14}\text{C}$. Thus $t = \dfrac{-\ln(0.29)}{1.209 \times 10^{-4} \text{ y}} = 1.0 \times 10^4 \text{ y}$.

**EVALUATE:** The half-life of ${}^{14}\text{C}$ is 5730 y, so our calculated *t* is about 1.75 half-lives, so the fraction remaining is around $\left(\tfrac{1}{2}\right)^{1.75} = 0.30$.

**43.74.** **IDENTIFY:** The tritium (H-3) decays to He-3. The ratio of the number of He-3 atoms to H-3 atoms allows us to calculate the time since the decay began, which is when the H-3 was formed by the nuclear explosion. The H-3 decay is exponential.

**SET UP:** The number of tritium (H-3) nuclei decreases exponentially as $N_\text{H} = N_{0,\text{H}} e^{-\lambda t}$, with a half-life of 12.3 years. The amount of He-3 present after a time *t* is equal to the original amount of tritium minus the number of tritium nuclei that are still undecayed after time *t*.

**EXECUTE:** The number of He-3 nuclei after time *t* is

$$N_\text{He} = N_{0,\text{H}} - N_\text{H} = N_{0,\text{H}} - N_{0,\text{H}} e^{-\lambda t} = N_{0,\text{H}}(1 - e^{-\lambda t}).$$

Taking the ratio of the number of He-3 atoms to the number of tritium (H-3) atoms gives

$$\frac{N_\text{He}}{N_\text{H}} = \frac{N_{0,\text{H}}(1 - e^{-\lambda t})}{N_{0,\text{H}} e^{-\lambda t}} = \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1.$$

Solving for *t* gives $t = \dfrac{\ln(1 + N_\text{He}/N_\text{H})}{\lambda}$. Using the given numbers and $T_{1/2} = \dfrac{\ln 2}{\lambda}$, we have

$\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{12.3 \text{ y}} = 0.0563/\text{y}$ and $t = \dfrac{\ln(1 + 4.3)}{0.0563/\text{y}} = 30$ years.

**EVALUATE:** One limitation on this method would be that after many years the ratio of H to He would be too small to measure accurately.

**43.75.** **(a) IDENTIFY** and **SET UP:** Use Eq. (43.1) to calculate the radius *R* of a ${}^{2}_{1}\text{H}$ nucleus. Calculate the Coulomb potential energy (Eq. 23.9) of the two nuclei when they just touch.

**EXECUTE:** The radius of ${}^{2}_{1}\text{H}$ is $R = (1.2 \times 10^{-15} \text{ m})(2)^{1/3} = 1.51 \times 10^{-15} \text{ m}$. The barrier energy is the Coulomb potential energy of two ${}^{2}_{1}\text{H}$ nuclei with their centers separated by twice this distance:

$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)\frac{(1.602 \times 10^{-19} \text{ C})^2}{2(1.51 \times 10^{-15} \text{ m})} = 7.64 \times 10^{-14} \text{ J} = 0.48 \text{ MeV}$$

**(b) IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease.

**EXECUTE:** ${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \to {}^{3}_{2}\text{He} + {}^{1}_{0}\text{n}$

If we use neutral atom masses there are two electrons on each side of the reaction equation, so their masses cancel. The neutral atom masses are given in Table 43.2.

${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H}$ has mass $2(2.014102\text{ u}) = 4.028204\text{ u}$

${}^{3}_{2}\text{He} + {}^{1}_{0}\text{n}$ has mass $3.016029\text{ u} + 1.008665\text{ u} = 4.024694\text{ u}$

The mass decrease is $4.028204\text{ u} - 4.024694\text{ u} = 3.510\times10^{-3}\text{ u}$. This corresponds to a liberated energy of $(3.510\times10^{-3}\text{ u})(931.5\text{ MeV/u}) = 3.270\text{ MeV}$, or $(3.270\times10^{6}\text{ eV})(1.602\times10^{-19}\text{ J/eV}) = 5.239\times10^{-13}\text{ J}$.

**(c) IDENTIFY** and **SET UP:** We know the energy released when two ${}^{2}_{1}\text{H}$ nuclei fuse. Find the number of reactions obtained with one mole of ${}^{2}_{1}\text{H}$.

**EXECUTE:** Each reaction takes two ${}^{2}_{1}\text{H}$ nuclei. Each mole of $D_2$ has $6.022\times10^{23}$ molecules, so $6.022\times10^{23}$ pairs of atoms. The energy liberated when one mole of deuterium undergoes fusion is $(6.022\times10^{23})(5.239\times10^{-13}\text{ J}) = 3.155\times10^{11}\text{ J/mol}$.

**EVALUATE:** The energy liberated per mole is more than a million times larger than from chemical combustion of one mole of hydrogen gas.

**43.76.** **IDENTIFY:** In terms of the number $\Delta N$ of cesium atoms that decay in one week and the mass $m = 1.0\text{ kg}$, the equivalent dose is $3.5\text{ Sv} = \dfrac{\Delta N}{m}((\text{RBE})_\gamma E_\gamma + (\text{RBE})_e E_e)$.

**SET UP:** $1\text{ day} = 8.64\times10^{4}\text{ s}$. $1\text{ year} = 3.156\times10^{7}\text{ s}$.

**EXECUTE:** $3.5\text{ Sv} = \dfrac{\Delta N}{m}((1)(0.66\text{ MeV}) + (1.5)(0.51\text{ MeV})) = \dfrac{\Delta N}{m}(2.283\times10^{-13}\text{ J})$, so

$$\Delta N = \frac{(1.0\text{ kg})(3.5\text{ Sv})}{(2.283\times10^{-13}\text{ J})} = 1.535\times10^{13}.\quad \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{(30.07\text{ y})(3.156\times10^{7}\text{ sec/y})} = 7.30\times10^{-10}\text{ sec}^{-1}.$$

$$\Delta N = |dN/dt|t = \lambda N t,\text{ so } N = \frac{\Delta N}{\lambda t} = \frac{1.535\times10^{13}}{(7.30\times10^{-10}\text{ s}^{-1})(7\text{ days})(8.64\times10^{4}\text{ s/day})} = 3.48\times10^{16}.$$

**EVALUATE:** We have assumed that $|dN/dt|$ is constant during a time of one week. That is a very good approximation, since the half-life is much greater than one week.

**43.77.** **IDENTIFY:** The speed of the center of mass is $v_{\text{cm}} = v\dfrac{m}{m+M}$, where $v$ is the speed of the colliding particle in the lab system. Let $K_{\text{cm}} \equiv K'$ be the kinetic energy in the center-of-mass system. $K'$ is calculated from the speed of each particle relative to the center of mass.

**SET UP:** Let $v'_m$ and $v'_M$ be the speeds of the two particles in the center-of-mass system. $Q$ is the reaction energy, as defined in Eq. (43.23). For an endoergic reaction, $Q$ is negative.

**EXECUTE:** **(a)** $v'_m = v - v\dfrac{m}{m+M} = \left(\dfrac{M}{m+M}\right)v$. $v'_M = \dfrac{vm}{m+M}$.

$$K' = \frac{1}{2}mv'^2_m + \frac{1}{2}Mv'^2_M = \frac{1}{2}\frac{mM^2}{(m+M)^2}v^2 + \frac{1}{2}\frac{Mm^2}{(m+M)^2}v^2 = \frac{1}{2}\frac{M}{(m+M)}\left(\frac{mM}{m+M} + \frac{m^2}{m+M}\right)v^2.$$

$$K' = \frac{M}{m+M}\left(\frac{1}{2}mv^2\right) \Rightarrow K' = \frac{M}{m+M}K \equiv K_{\text{cm}}.$$

**(b)** For an endoergic reaction $K_{\text{cm}} = -Q(Q < 0)$ at threshold. Putting this into part (a) gives

$$-Q = \frac{M}{M+m}K_{\text{th}} \Rightarrow K_{\text{th}} = \frac{-(M+m)}{M}Q.$$

**EVALUATE:** For $m = M$, $K' = K/2$. In this case, only half the kinetic energy of the colliding particle, as measured in the lab, is available to the reaction. Conservation of linear momentum requires that half of $K$ be retained as translational kinetic energy.

**43.78. IDENTIFY** and **SET UP:** Calculate the energy equivalent of the mass decrease.

**EXECUTE:** $\Delta m = M({}^{235}_{92}\text{U}) - M({}^{140}_{54}\text{Xe}) - M({}^{94}_{38}\text{Sr}) - m_{\text{n}}$

$\Delta m = 235.043923\text{ u} - 139.921636\text{ u} - 93.915360\text{ u} - 1.008665\text{ u} = 0.1983\text{ u}$

$\Rightarrow E = (\Delta m)c^2 = (0.1983\text{ u})(931.5\text{ MeV/u}) = 185\text{ MeV}.$

**EVALUATE:** The calculation with neutral atom masses includes 92 electrons on each side of the reaction equation, so the electron masses cancel.

**43.79. IDENTIFY** and **SET UP:** $\left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t}$ for each species. $\ln|dN/dt| = \ln(\lambda N_0) - \lambda t$. The longer-lived nuclide dominates the activity for the larger values of $t$ and when this is the case a plot of $\ln|dN/dt|$ versus $t$ gives a straight line with slope $-\lambda$.

**EXECUTE:** **(a)** A least-squares fit of the log of the activity *vs.* time for the times later than 4.0 h gives a fit with correlation $-(1 - 2\times10^{-6})$ and decay constant of $0.361\text{ h}^{-1}$, corresponding to a half-life of 1.92 h. Extrapolating this back to time 0 gives a contribution to the rate of about 2500/s for this longer-lived species. A least-squares fit of the log of the activity *vs.* time for times earlier than 2.0 h gives a fit with correlation = 0.994, indicating the presence of only two species.

**(b)** By trial and error, the data is fit by a decay rate modeled by $R = (5000\text{ Bq})e^{-t(1.733/\text{h})} + (2500\text{ Bq})e^{-t(0.361/\text{h})}$. This would correspond to half-lives of $0.400\text{ h}$ and $1.92\text{ h}$.

**(c)** In this model, there are $1.04\times10^7$ of the shorter-lived species and $2.49\times10^7$ of the longer-lived species.

**(d)** After $5.0\text{ h}$, there would be $1.80\times10^3$ of the shorter-lived species and $4.10\times10^6$ of the longer-lived species.

**EVALUATE:** After $5.0\text{ h}$, the number of shorter-lived nuclei is much less than the number of longer-lived nuclei.

**43.80. IDENTIFY:** Apply $A = A_0 e^{-\lambda t}$, where $A$ is the activity and $\lambda = \frac{\ln 2}{T_{1/2}}$. This equation can be written as $A = A_0 2^{-(t/T_{1/2})}$. The activity of the engine oil is proportional to the mass worn from the piston rings.

**SET UP:** $1\text{ Ci} = 3.7\times10^{10}\text{ Bq}$

**EXECUTE:** The activity of the original iron, after 1000 hours of operation, would be $(9.4\times10^{-6}\text{ Ci})(3.7\times10^{10}\text{ Bq/Ci})2^{-(1000\text{ h})/(45\text{ d}\times24\text{ h/d})} = 1.8306\times10^5$ Bq. The activity of the oil is 84 Bq, or $4.5886\times10^{-4}$ of the total iron activity, and this must be the fraction of the mass worn, or mass of $4.59\times10^{-2}$ g. The rate at which the piston rings lost their mass is then $4.59\times10^{-5}$ g/h.

**EVALUATE:** This method is very sensitive and can measure very small amounts of wear.